RESEARCH ARTICLE

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Limit of axial force ratio and requirement for stirrups of RC columns with special shape

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Abstract Thousands of columns with special shape are analyzed by nonlinear numerical methods. The ductility is calculated to investigate the limit of the axial force ratio and circumstantial requirement for stirrups of an reinforced concrete (RC) column with special shape, in the point of view of the characteristic value for providing stirrup. The limit of the axial force ratio of columns with special shape in relation to the characteristic value of the stirrup is obtained. Then, the effect of stirrup arrangement on the ductility of the RC column is discussed in case of buckling of the longitudinal reinforcement and constraint concrete columns. The complete requirement for stirrups of RC column with special shape is given.

Keywords column with special shape, the limit of axial force ratio, ductility, spacing of stirrups, characteristic value for providing stirrup

1 Introduction

The studies of Refs. [1-4] show that the axial force ratio, angle of moment actions, and the ratio of spacing of stirrups to the diameter of the reinforcements, s/d, affect the ductility of columns with special shapes. The value s/d related to the limited value of the axial force ratio was presented in Ref. [1], and the detailed requirements for the stirrup of columns with special shapes took into consideration the stipulations of the rectangle shaped columns defined in the code for the design of concrete structures of China [5].

The limit of the axial force ratio is related to the characteristic value for providing stirrups, in the code of China and Europe [6]. In order to hold an identical rule with the national code, the study of the limit of the axial force ratio of column with special shape is required from the point of view of the characteristic value to provide stirrups.

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WANG Yiqun (🖾), XU Yidong, ZHAO Yanjing, CHEN Yunxia School of Civil Engineering, Tianjin University, Tianjin 300072, China E-mail: yqwangtj@hotmail.com Comparing the columns with the special shape to rectangular columns in terms of ductility and the bearing capability, many large differences can be identified. Therefore, a thorough requirement for the stirrup must be researched based on the properties of column with special shape itself. Thousands of sets of columns are analyzed in this paper to find the relationship between the value of axial force ratio and the characteristic value for providing stirrup in various earthquake-resistant grades. Furthermore, based on the computed limit of the axial force ratio, the influence of the spacing of stirrups *s* and *s/d* on the ductility of column with special shape is found, and the detailed requirement for stirrups of column with special shape is demonstrated.

2 Limit of axial force ratio for a column with special shape

2.1 Relationship between axial force ratio and characteristic value for providing stirrup

2.1.1 Computer program

A computer program was developed based on the work in Ref. [1], and some changes are made as follows.

First, the ultimate curvature is set to the section curvature at buckling of the compressive steel reinforcement or the moment M dropping to 0.85 M_{max} .

Second, tie bars are considered in the ratio of volumetric reinforcement for stirrups.

Third, a modified Kent-Park model is adopted for the stress-strain relationship of the compression zone of concrete (see Fig. 1).

Region AB ($\varepsilon_{\rm c} \leq k\varepsilon_0$)

$$\sigma_{\rm c} = k f_{\rm c} \left[\frac{2\varepsilon_{\rm c}}{k\varepsilon_0} - \left(\frac{\varepsilon_{\rm c}}{k\varepsilon_0} \right)^2 \right] \tag{1}$$

Region BC ($\varepsilon_{\rm c} > k\varepsilon_0$)

$$\sigma_{\rm c} = k f_{\rm c} \left[1 - Z_{\rm m} (\varepsilon_{\rm c} - k \varepsilon_0) \right], \ \sigma_{\rm c} \ge 0.2 k f_{\rm c} \tag{2}$$

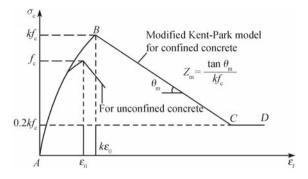


Fig. 1 Stress-strain curve for confined concrete

in which

$$Z_{\rm m} = \frac{0.5}{\frac{3+0.29f_{\rm c}'}{145f_{\rm c}'-1000} + 0.75\rho_{\rm sv}\sqrt{\frac{h_{\rm c}}{s_{\rm h}}} - 0.002k}$$
(3)
$$k = 1 + \rho_{\rm sv}f_{\rm yv}/f_{\rm c}'$$
(4)

Here, k is the enhancement coefficient of strength for concrete confined by stirrups; ε_0 is the peak value compressive strain for concrete corresponding to f'_c , 0.002 is adopted; f'_c is the compressive cylinder strength, $f'_c = 0.80f_{cu}$ is used in the paper; f_{cu} is the compressive strength of the 150-mm side length concrete cube used in China; ρ_{sv} is the ratio of volumetric reinforcement for the stirrup; h_c is the width of the concrete confined by stirrups; s_h is the spacing of stirrups; f_{yv} is the value of yield strength of the stirrup.

2.1.2 Comparisons between calculated results and test results

The specimens in Refs. [9–11] were calculated using the program developed by us. Comparison between calculated values and test results are shown in Table 1. The table shows that the ductility of curvature calculated by the program is in a good agreement with the test results.

 Table 1
 Comparisons between calculation and test results

Section shape	No. of specimen	μ_{arphi} by test	μ_{φ} calculated	Error/%	Angle of moment actions/(°)
L[9]	Z-4	3.59	3.86	-7.52	135
R[10]	Z-6	6.42	6.07	5.45	15
+[10]	Z-7	5.90	5.95	-0.85	0
	Z-8	6.41	6.66	-3.90	15
T[11]	No.4	4.81	5.15	-7.07	22.5
	No.9	7.49	8.55	-14.15	67.5
	No.10	5.28	5.57	-5.49	90.0

Note: R for Rectangle

2.1.3 Calculation parameters

A serial L, T and + shaped section of columns with equal length legs are chosen for calculation analysis. These are

200 mm × 500 mm, 200 mm × 600 mm, 200 mm × 700 mm, 200 mm × 800 mm, 250 mm × 800 mm sections, where the two numbers are the thickness and height of the column leg, respectively. Concrete strength grade are in the range C30–C50. Diameters of stirrup (HPB235) d_v are 6 mm, 8 mm, and 10 mm respectively. Spacing of stirrups is in the range 70–150 mm respectively. Diameters of longitudinal reinforcement (HRB335) are in the range of 16–25 mm. All the detailing requirements about the longitudinal reinforcements, stirrups, and tie bars are in keeping with the specifications in Ref. [12].

2.1.4 Analysis of calculation results

First, the unfavorable moment action angle zones of L, T, and + shaped section columns under different axial force ratios are determined. Second, the curvature ductility ratio μ_{φ} of L, T, and + shaped section columns in the unfavorable angle zones are calculated. Then the relationship between the curvature ductility ratio μ_{φ} , characteristic value for providing stirrup λ_{v} , and characteristic value of axial force ratio n_{k} are gotten by regression.

The column curvature ductility ratio μ_{φ} are set to 9–10, 7–8, 5–6 [13] for the earthquake-resistant grade II, III, and IV, respectively. And using the relationship, design value of axial force ratio $n = N/f_cA = 1.2N_k/[f_{ck}/1.40A] = 1.68n_k$, the λ_v -*n* curves can be obtained as Fig. 2.

2.2 Limit of axial force ratio and minimum characteristic value for providing stirrup or hoop in the densified zone of a column with special shape in various earthquake-resistant grade

Obviously, in order to stipulate the limited value of the axial force ratio, the maximum value of the characteristic for providing stirrup $\lambda_{v,max}$ in construction must be determined. The characteristic value for providing stirrup λ_v is related with the ratio of volumetric reinforcement for stirrup in densified zone of column with special shape, ρ_v , design value of axial compressive strength of concrete, f_c , and design value of tensile strength of stirrups or tie bars, f_{vv} , that is

$$\lambda_{\rm v} = \frac{f_{\rm yv}}{f_{\rm c}} \rho_{\rm v}$$

By the formula above, it is shown that $\lambda_{v,max}$ depends upon the maximum value of the ratio of volumetric reinforcement for stirrup $\rho_{v,max}$, in a synthetical study of the influences of concrete strength, stirrup, and tie bars, etc. Given an example, L200 mm × 700 mm, strength grade of concrete C40, stirrup of HPB235, for this case, $\lambda_v = 10.995\rho_v$. If the stirrup of $\phi 10@100$ is adopted, then $\rho_v = 0.0164$, $\lambda_v = 0.18$; If the stirrup of $\phi 10@90$ is adopted, then $\rho_v = 0.0182$, $\lambda_v = 0.20$. Considering the section dimension of L-shaped columns, the maximum of characteristic value for providing stirrup for L-shaped columns is 0.18-0.20; $\lambda_{v,max}$ for T-shaped columns is 0.19-0.21; $\lambda_{v,max}$ for +"-shaped columns is 0.20-0.22, in the

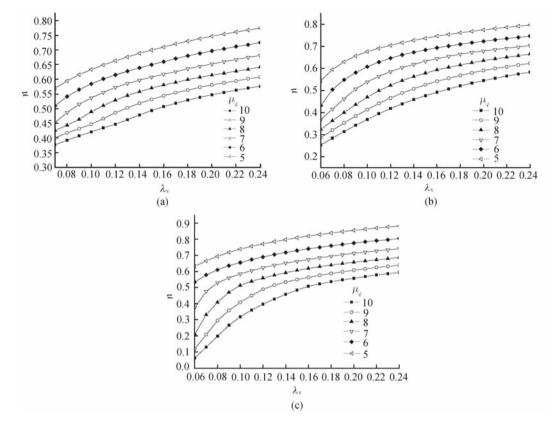


Fig. 2 Relation of axial force ratio of columns with special shape to characteristic value for providing stirrup or hoop (a) L-shaped section columns; (b) T-shaped section columns; (c) +-shaped section columns

same reason. So, according to Fig. 2, the limited value of axial force ratio for columns with special shape in different earthquake-resistant grades is obtained, as shown in Table 2. Based on Table 2, and through appropriate readjustment, the minimum of characteristics for providing stirrup in the densified zone of column with special shape is obtained, as shown in Table 3.

2.3 Limit of axial force ratio for columns with special shape and with unequal legs

According to the results of computation, the limited value of axial force ratio for columns with special shape with unequal

Table 2 Limit of axial force ratio for columns with special shape

System of structures	Section shape	Earthquake-resistant grade			
		II	III	IV	
Frame structure	L	0.50	0.60	0.70	
	Т	0.55	0.65	0.75	
	+	0.60	0.70	0.80	
Frame-structural(shear)	L	0.55	0.65	0.75	
wall structure	Т	0.60	0.70	0.80	
	+	0.65	0.75	0.85	

legs also can be stipulated with the same values in Table 2. Taking the L-shaped column with unequal legs for example,

Table 3 Minimum characteristic value for providing stirrup or hoop in densified zone of columns with special shape

Section	Earthquake-resistant		Value of axial force ratio									
shape	grade	≤0.30	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
L-shaped	II	0.10	0.13	0.15	0.18	0.20						
-	III	0.09	0.10	0.12	0.14	0.16	0.18	0.20				
	IV	0.08	0.09	0.10	0.11	0.12	0.14	0.16	0.18	0.20		
T-shaped	II	0.09	0.12	0.14	0.17	0.19	0.21					
	III	0.08	0.09	0.11	0.13	0.15	0.17	0.19	0.21			
	IV	0.07	0.08	0.09	0.10	0.11	0.13	0.15	0.17	0.19	0.21	
+-shaped	II	0.08	0.11	0.13	0.16	0.18	0.20	0.22				
	III	0.07	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22		
	IV	0.06	0.07	0.08	0.09	0.10	0.12	0.14	0.16	0.18	0.20	0.22

calculation analysis about some sections, such as 200 mm × 500 mm × 600 mm, 200 mm × 500 mm × 800 mm, 200 mm × 600 mm × 700 mm, 250 mm × 600 mm × 800 mm (the three numbers indicate the thickness of the legs and the lengths of the two legs respectively), the $\lambda_v - n$ relationship curves are obtained, shown in Fig. 3, for earthquake-resistant grade II. From Fig. 3 we can see that, compared with the case of the L-shaped column with equal legs, the curves are only fluctuating minimally about the curve of the equal legs, and shows good consistency. Therefore, the stipulations in Tables 2 and 3 are applicable for the columns of special shape with unequal legs, wherein the ratio of longer leg to shorter leg is in the range listed above.

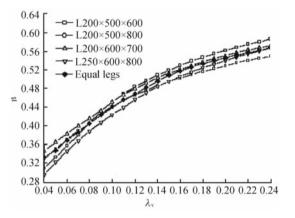


Fig. 3 λ_v -*n* relationship of L-shaped columns with equal and unequal legs

3 Rational dispositions of stirrups for columns with special shapes

3.1 Influence of *s/d* to buckling of longitudinal reinforcement of columns with special shape

Statistical analyses are made for the buckling of longitudinal reinforcement under the maximum axial force ratio for different earthquake-resistant grades.

3.1.1 L-shaped columns

1. For the case of maximum axial force ratio n = 0.50, $\mu_{\varphi} = 9 \sim 10$ (corresponding to earthquake-resistant grade II), the buckling of the longitudinal reinforcement is shown in Table 4 wherein the number of columns with buckled longitudinal bars is the number of columns with ultimate curvature controlled by buckling of longitudinal reinforcement. The total number of columns is 4320 for the columns that are in keeping with relevant earthquake-resistant grades.

2. For the case of maximum axial force ratio n = 0.60, $\mu_{\varphi} = 7 \sim 8$ (corresponding to earthquake-resistant grade III), the buckling of the longitudinal reinforcement did not occur.

 Table 4
 Statistics for buckling of longitudinal reinforcement of L-shaped columns for earthquake- resistant grade-II

Range of <i>s/d</i>	$N_{ m L}$	$N_{ m T}$	$N_{\rm L}/N_{\rm T}$ / %
[4,5]	3		0.08
[5,6]	7	3837	0.18
[6,7]	13		0.34

Note: $N_{\rm L}$ denotes number of columns with buckled longitudinal bars; $N_{\rm T}$ denotes total number of columns.

3. For the case of the maximum axial force ratio n = 0.70, $\mu_{\varphi} = 5 \sim 6$ (corresponding to earthquake- resistant grade IV), buckling of the longitudinal reinforcement did not occur.

3.1.2 Columns of T-shape and +-shape

Buckling of longitudinal reinforcement did not occur in different earthquake-resistant grades.

3.1.3 Results analyses

From Sect. 3.1.1, we can see that, as the demanding curvature ductility μ_{φ} is reduced, the percent of the number of columns with buckling reinforcement to total number of columns is decreased. The percent is 0.34 for earthquake-resistant grade II, and 0.00 for earthquake-resistant grade III, and IV, and with the increasing of s/d, the percent is increased. With the increase of the value of s/d, the non-support length of longitudinal reinforcement increases, and it accelerates the buckling of the longitudinally reinforced column. Thereby, the ultimate curvature φ_u is decreased, and the ductility of the column is reduced. So, the value of s/d must be controlled in order to increase the ductility of columns.

The percent of longitudinal reinforcement buckling is small for the earthquake-resistant grade II, based upon the analyses above. The value of s/d is suggested as 6 for earthquake-resistant grade II, so the percent of longitudinal reinforcement buckling can be controlled under 0.34. The value of s/d is suggested as 7 for earthquake-resistant grade III, and IV. Moreover, considering the confining action of stirrups to concrete, the spacing of stirrups *s* must be restricted. According analysis and to Ref. [14], the maximum spacing of stirrups shall not be greater than 100 mm, 120 mm, and 150 mm for earthquake-resistant grade II, III, and IV, respectively. And, the maximum spacing of stirrups at column root must not be greater than 100 mm.

The limit of s/d for columns of T-shape and +-shape in different earthquake-resistant grade are the same for columns of L-shape.

3.2 Rational dispositions for *s*, d_v and ρ_v

When the ratios of reinforcement for stirrup ρ_v are the same, but the diameter of stirrup d_v and spacing of stirrup *s* are not equal, the parameter and calculated results are listed in Tables 5 and 6, respectively.

Table 5 Calculating parameters with same ρ_v of L-shaped columns

No.	Dimensions / mm × mm	Value of axial force ratio	Strength grade of concrete	Diameter of longitudinal reinforcement/mm	Dispositions of stirrups	ratio of volumetric reinforcement for stirrup /%	Angle of moment action/(°)
1-1a 1-1b	200×700	0.50	C30	18	$\phi 8@80 \\ \phi 10@125$	1.31	247.5
1-2a 1-2b	200×700	0.60	C35	25	$\phi 8@100 \\ \phi 10@150$	1.06	237.5
1-3a 1-3b	200×600	0.50	C30	20	φ8@80 φ10@125	1.31	247.5

Table 6	Calculated results with same ρ_v of L-shaped columns
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No.	Control condition for failure	Yield curvature φ_{y}	Ultimate curvature φ_u	Ratio of ductility of curvature μ_{φ}	$\frac{\mu_{\rm a\phi}-\mu_{\rm b\phi}}{\mu_{\rm b\phi}}{\times}100\%$
1-1a	Buckling of longitudinal reinforcement	0.00546	0.09827	17.9980	13.63
1-1b		0.00547	0.08664	15.8390	
1-2a	$0.85M_{\rm max}$	0.00660	0.05817	8.8136	7.01
1-2b		0.00660	0.05436	8.2364	
1-3a	Buckling of longitudinal reinforcement	0.00648	0.11435	17.6466	14.34
1-3b	$0.85M_{\rm max}$	0.00649	0.10016	15.4330	

There are 3 groups of specimen. In each group, there are 2 columns with different stirrup dispositions and with the same $\varphi_{\rm v}$, but they are with different ultimate curvature $\varphi_{\rm u}$. That is, for the columns with smaller spacing of stirrups s, the φ_{μ} is increased, therefore, the ratio of curvature ductility is increased. From Table 6, we can see that the φ_{μ} of specimen 1-1a, 1-2a, and 1-3a increased by 11.63%, 7.01%, and 14.34% from specimen 1-1b, 1-2b, and 1-3b, respectively. This happens if the ratios of volumetric reinforcement for the stirrup are equal, the ductility of columns with dispositions of smaller spacing of stirrups s and smaller diameter of stirrup $d_{\rm v}$ is larger than that with dispositions of bigger spacing of stirrups s and larger diameter of stirrup d_{y} , whether it is controlled by the buckling of longitudinal reinforcement or M decreases to $0.85M_{\text{max}}$. The reason is that the smaller spacing of stirrups does not only reduce the possibility of buckling of the longitudinal reinforcement, but also make the descending

segment of the stress-strain curve of concrete more gentle, and make the ultimate value of deformation larger. Therefore, when φ_{u} increases, the ductility also increases.

When the ratios of reinforcement for stirrup ρ_v are not the same, the parameter and calculated results are listed in Tables 7 and 8, respectively.

Form Table 8, we can see that in the 2 groups of specimen with great difference of *s*, specimens 1-4b and 1-5b are with larger d_v , and their ratios of volumetric reinforcement for stirrup ρ_u is increased by 29.52% and 3.82%, respectively. Because the *s* is larger, the confining effectiveness of stirrups on the concrete and longitudinal reinforcement is decreased, and it cannot increase the ultimate curvature φ_u effectively. Therefore, the ratios of curvature ductility of these are decreased by 4.23% and 6.86% compared to specimen 1-4a and 1-5a. From this we can see that the ductility of column with special shape cannot be increased by adopting a larger

Table 7 Calculation parameters with different ρ_v of L-shaped columns

No.	Dimensions / mm × mm	Value of axial force ratio	Strength grade of concrete	Diameter of longitudinal reinforcement/mm	Dispositions of stirrups	Ratio of volumetric reinforcement for stirrup		Angle of moment action/(°)
						$ ho_{ m v}$ /%	Increased /%	
1-4a 1-4b	200×500	0.50	C30	25	$\phi 8@100 \\ \phi 10@120$	1.05 1.36	29.52	247.5
1-5a 1-5b	200×800	0.60	C40	20	φ8@80 φ10@120	1.31 1.36	3.82	57.5

Table 8	Calculation	results with	different	$\rho_{\rm v}$ of L	l-shaped	columns
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No.	Control condition for failure	Yield curvature φ_y	Ultimate curvature φ_u	Ratio of ductility of curvature μ_{φ}	$\frac{\mu_{\rm ap}-\mu_{\rm bp}}{\mu_{\rm bp}}{\times}100\%$
1-4a	Buckling of longitudinal reinforcement	0.008	0.130	16.382	4.23
1-4b		0.008	0.126	15.717	
1-5a	$0.85M_{\rm max}$	0.005	0.055	10.028	6.86
1-5b		0.005	0.051	9.384	

diameter of stirrup to raise its volumetric reinforcement, and not decrease the spacing of the stirrups at same time. In order to increase the ductility of column with special shape, the ratio of volumetric reinforcement for stirrup ρ_v must be increased with the spacing of stirrups *s* satisfying the requirements of support lengths of longitudinal reinforcement.

The study in Ref. [15] showed that, following the increase of stirrup diameter, the rigidity of the stirrups for avoiding the longitudinal steel bars been pushed outside increased, and the confining pressure on concrete also increased. Therefore, in order to make the most of the confining effect of stirrup to concrete, some requirements on the diameter of the stirrup should be made, in addition to restricting the spacing of stirrup *s*. Considering the features of column with special shape, and referring to Ref. [14], the minimum diameter of the stirrup for columns with special shape are presented, as shown in Table 9.

 Table 9
 Detailing requirements for densified zone of stirrups at ends of columns with special shape

Earthquake- resistant grade	Maximum spacing of stirrups/mm	Minimum diameter of stirrup/mm
II	Smaller value between 6 times of	8
	diameter of longitudinal steel bar and 100	
III	Smaller value between 7 times	8
	of diameter of longitudinal steel bar	
	and 120 (100 at column root)	
IV	Smaller value between 7 times of	6 (8 at
	diameter of longitudinal steel bar	column root)
	and 150 (100 at column root)	

In order to ensure the ductility of the column of special shape, the dispositions of the stirrups should satisfy the requirements of Tables 3 and 9, simultaneously.

4 Conclusions

First, the axial force ratio limit and the minimum characteristics for the stirrup of different earthquake-resistant grade are presented in the paper, based on computational analyses.

Second, the ratio s/d, or the spacing of stirrups *s* to the diameter of longitudinal steel bar *d*, directly affects buckling of longitudinally reinforced steel bars. An increase of the value of s/d would accelerate buckling of the steel bars. On the other hand, a decrease would delay buckling of the steel bars, and increase the ductility of the column.

Third, when the ratios of circumstantial reinforcement for the stirrup are the same, the arrangement using stirrups of small diameter and close spacing is better than the one using stirrups of large diameter and wide spacing. Compared with the cases of different spacing and diameter of stirrups, the ductility of the special type of columns is not greater because of the large value of the ratios of circumstantial reinforcement for the stirrup. With rational arrangement of the spacing of stirrups *s*, and increase of the ratio of circumstantial reinforcement for the stirrup, the ductility of the column can be increased efficiently.

Fourth, detailed requirements for densified zone of the column of special shape are presented, on the basis of the dispositions analysis of the stirrups.

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